Rome - December, 20 2011

Spectrum Management and Cognitive Radios

Alessandro Guidotti

Ph.D. student – Electronics, Computer Science and Telecommunications

Tutor: Prof. Ing. Giovanni Emanuele Corazza, University of Bologna

Cotutor: Ing. Guido Riva, Fondazione Ugo Bordoni

3 years have gone…

- Mutual interference between DVB-T and LTE systems
- Quantification of the amount of spectrum potentially available as *white space*
- Issues related to geolocation databases
- Characterization of the Hidden Node Margin
- Analysis of cooperative energy detection algorithms
- Coexistence between WSD and TETRA systems
- Cognitive Hybrid Satellite Terrestrial Radios
- Stochastic Geometry theory for network interference characterization (SUPELEC, Paris)

Stochastic Geometry: why?

- Network interference is a key issue in 4G systems
- Three factors to be considered
  - Spatial distribution of the nodes
  - Transmission characteristics
  - Propagation characteristics
- New *spatial average* performance metrics are needed:

Stochastic Geometry models → **Poisson Point Processes**

PPP: a collection of locations on the d-dimensional Euclidean plane, deployed according to a Poisson distribution

Objective and system model

- Evaluation of the Moment Generating Function of the aggregate interference from PPP-modeled BSs

$$\mathcal{M}_{\mathcal{I}_\Phi}(s) = \mathbb{E}_{\mathcal{I}_\Phi}\left\{e^{-s\mathcal{I}_\Phi}\right\} = \mathbb{E}_{q,b,|\tilde{h}|^2,\|\mathbf{x}\|}\left\{e^{-s\mathcal{I}_\Phi}\right\}$$

  - The intended user is connected with the closest one
  - All the other BSs act as interferers
  - Metrics: coverage probability, average rate, ASEP

- *Slyvniak's Theorem*: the characteristics of a PPP are not affected by the measurement point

$$\mathcal{I}_\Phi = \sum_{\mathbf{x}_i \in \Phi} q_i b_i |h_i|^2 = \sum_{\mathbf{x}_i \in \Phi} q_i b_i |\tilde{h}_i|^2 \ell(\|\mathbf{x}_i\|) = \sum_{\mathbf{x}_i \in \Phi} q_i b_i |\tilde{h}_i|^2 \|\mathbf{x}_i\|^{-\alpha}$$

$$\ell(\|\mathbf{x}_i\|) = R_i^{-\alpha} \quad \alpha > d$$

Moment Generating Function

$$\mathcal{M}_{\mathcal{I}_\Phi}(s) = \prod_{j=1}^{M} \exp\left\{p\lambda c_d R_1^d \mathbb{E}_{|\tilde{h}|^2}\left\{1 - e^{-sb_j|\tilde{h}|^2 R_1^{-\alpha}}\right\} - p\lambda c_d R_2^d \mathbb{E}_{|\tilde{h}|^2}\left\{1 - e^{-sb_j|\tilde{h}|^2 R_2^{-\alpha}}\right\}\right.$$

$$+ p\lambda c_d (sb_j)^{d/\alpha} \mathbb{E}_{|\tilde{h}|^2}\left\{|\tilde{h}|^{2d/\alpha}\Gamma\left(1 - \frac{d}{\alpha}, sb_j|\tilde{h}|^2 R_1^{-\alpha}\right)\right\}$$

$$\left. - p\lambda c_d (sb_j)^{d/\alpha} \mathbb{E}_{|\tilde{h}|^2}\left\{|\tilde{h}|^{2d/\alpha}\Gamma\left(1 - \frac{d}{\alpha}, sb_j|\tilde{h}|^2 R_2^{-\alpha}\right)\right\}\right\}^{\frac{1}{M}}$$

- Previous works
  - Upper bound approximations
  - Only Rayleigh and non fading channels
  - d=2 and α=2
  - Infinite networks

Moment Generating Function

- Our result:

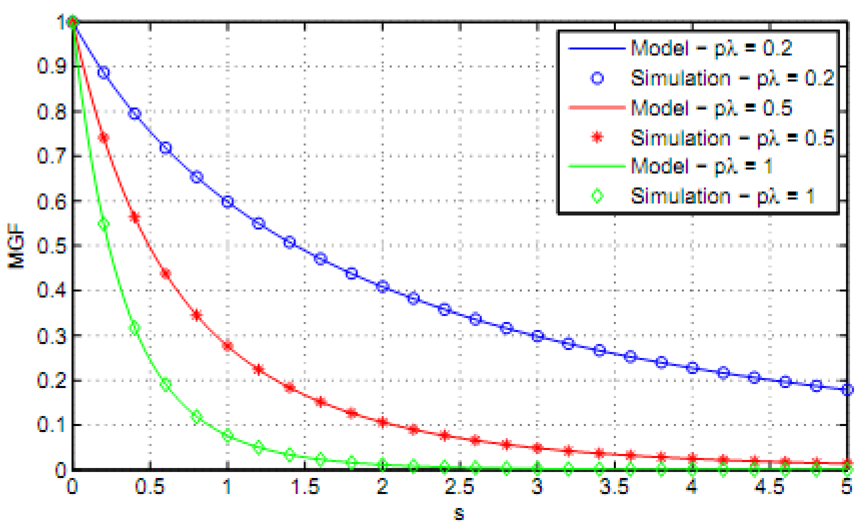
$$\mathcal{M}_{\mathcal{I}_\Phi}(s) = \exp\left[pc_d\lambda R_1^d - pc_d\lambda R_2^d + \frac{pc_d\lambda R_2^d}{\left(1+\frac{sR_2^{-\alpha}}{m_0}\right)^{m_0}} - \frac{pc_d\lambda R_1^d}{\left(1+\frac{sR_1^{-\alpha}}{m_0}\right)^{m_0}}\right]$$

$$\times \exp\left[-pc_d\lambda s R_1^{(d-\alpha)} \frac{m_0^{m_0}\Gamma(1+m_0)}{\Gamma(m_0)\left(1-\frac{d}{\alpha}\right)} \frac{{}_2F_1\left(m_0+1;1;2-\frac{d}{\alpha};\frac{s}{s+m_0R_1^\alpha}\right)}{\left(m_0+sR_1^{-\alpha}\right)^{(m_0+1)}}\right]$$

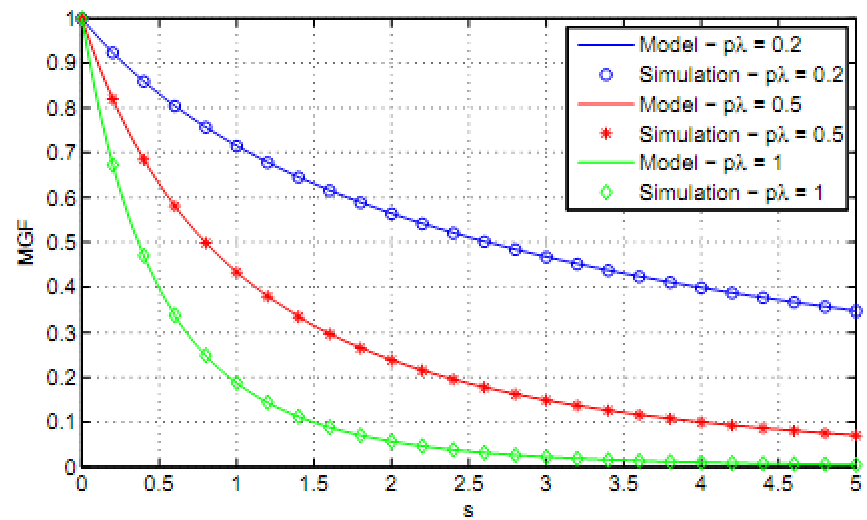
$$\times \exp\left[pc_d\lambda s R_2^{(d-\alpha)} \frac{m_0^{m_0}\Gamma(1+m_0)}{\Gamma(m_0)\left(1-\frac{d}{\alpha}\right)} \frac{{}_2F_1\left(m_0+1;1;2-\frac{d}{\alpha};\frac{s}{s+m_0R_2^\alpha}\right)}{\left(m_0+sR_2^{-\alpha}\right)^{(m_0+1)}}\right]$$

- Generalized fading channels
- d-dimensional networks
- Any network shape

Moment Generating Function



(a) $\alpha = 4$



(b) $\alpha = 5$

- Nakagami-m fading, m_0=2, R_1=1, R_2=32

Average Rate

- Andrews et al.: three-fold integral, useful only for α>4
- Our result: single-integral expression with a Meijer-G function
  - Obtained with Meijer-G function, hypergeometric function, series expansion of the upper incomplete Gamma function
  - Valid for α>2 and for generalized fading channels
  - Different fading models on intended and interfering links

$$\mathcal{R}(\lambda_{BS},\alpha)=\pi\lambda_{BS}\int_0^\infty\left[\frac{\mathcal{F}_1(y)}{\mathcal{F}_2(y)}-\frac{\alpha y\sqrt{k_\alpha l_\alpha^{(\alpha-1)}}\mathcal{F}_1(y)}{2(2\pi)^{\left(\frac{l_\alpha+k_\alpha}{2}-1\right)}[\mathcal{F}_2(y)]^{\left(\frac{\alpha}{2}+1\right)}}\mathcal{U}_{rate}(y)\right]\mathrm{d}y$$

Upper Bound

$$\mathcal{U}_{rate}(y)=G_{l_\alpha,k_\alpha}^{k_\alpha,l_\alpha}\left(\frac{y^{k_\alpha}l_\alpha^{l_\alpha}}{\mathcal{F}_2^{l_\alpha}(y)k_\alpha^{k_\alpha}}\middle|\begin{matrix}\mathbf{v}\\\mathbf{w}\end{matrix}\right)$$

Average Rate

- Nakagami-m fading

$$\mathcal{F}_{1,Nak}(y) = \frac{1}{y}\left[1-\left(1+\frac{y}{m_0}\gamma_0\right)^{-m_0}\right]$$

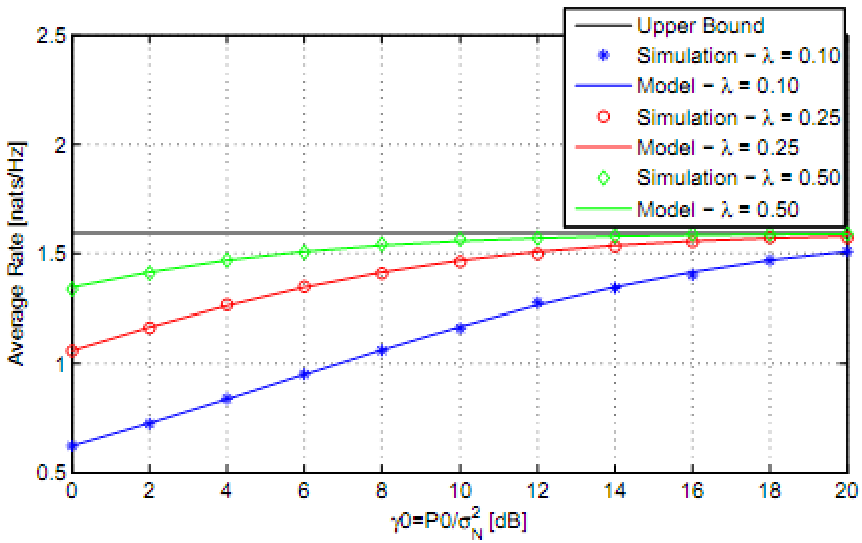
$$\mathcal{F}_{2,Nak}(y) = \frac{\pi\lambda_{BS}}{\left(1+\frac{y\gamma_I}{m_I}\right)^{m_I}} + \pi\lambda_{BS} y\gamma_I {m_I}^{(m_I+1)} \frac{{}_2F_1\left(m_I+1;1;2-\frac{2}{\alpha};\frac{y\gamma_I}{y\gamma_I+m_I}\right)}{\left(1-\frac{2}{\alpha}\right)\left(m_I+y\gamma_I\right)^{(m_I+1)}}$$

- Rayleigh fading

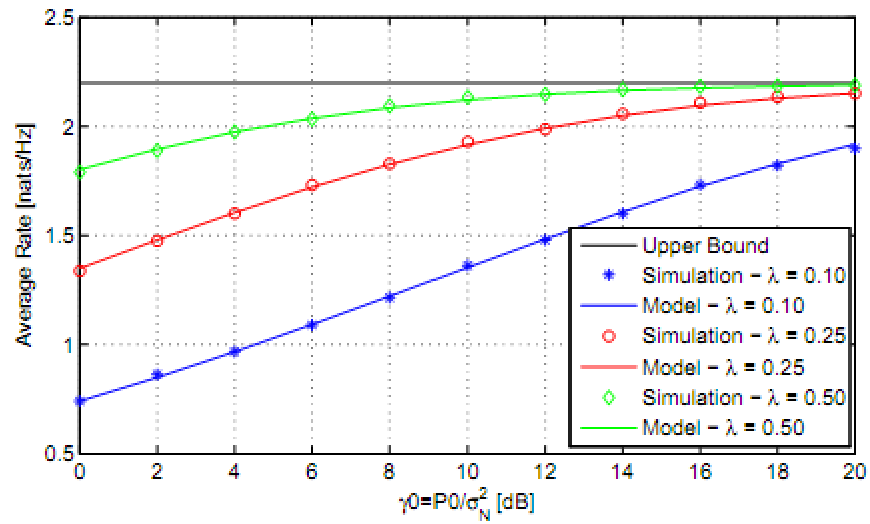
$$\mathcal{F}_{1,Ray}(y) = \frac{1}{y}\left[1-\left(1+y\gamma_0\right)^{-1}\right]$$

$$\mathcal{F}_2(y) = \frac{\pi\lambda_{BS}}{1+\gamma_I y} + \pi\lambda_{BS} y\gamma_I \frac{{}_2F_1\left(2;1;2-\frac{2}{\alpha};\frac{y\gamma_I}{y\gamma_I+1}\right)}{\left(1-\frac{2}{\alpha}\right)\left(1+y\gamma_I\right)^2}$$

Average Rate



(a) $\alpha = 4$



(b) $\alpha = 5$

- Intended link with Nakagami-m fading, $m_0=2$
- Interfering link with Rayleigh fading
- Excellent agreement

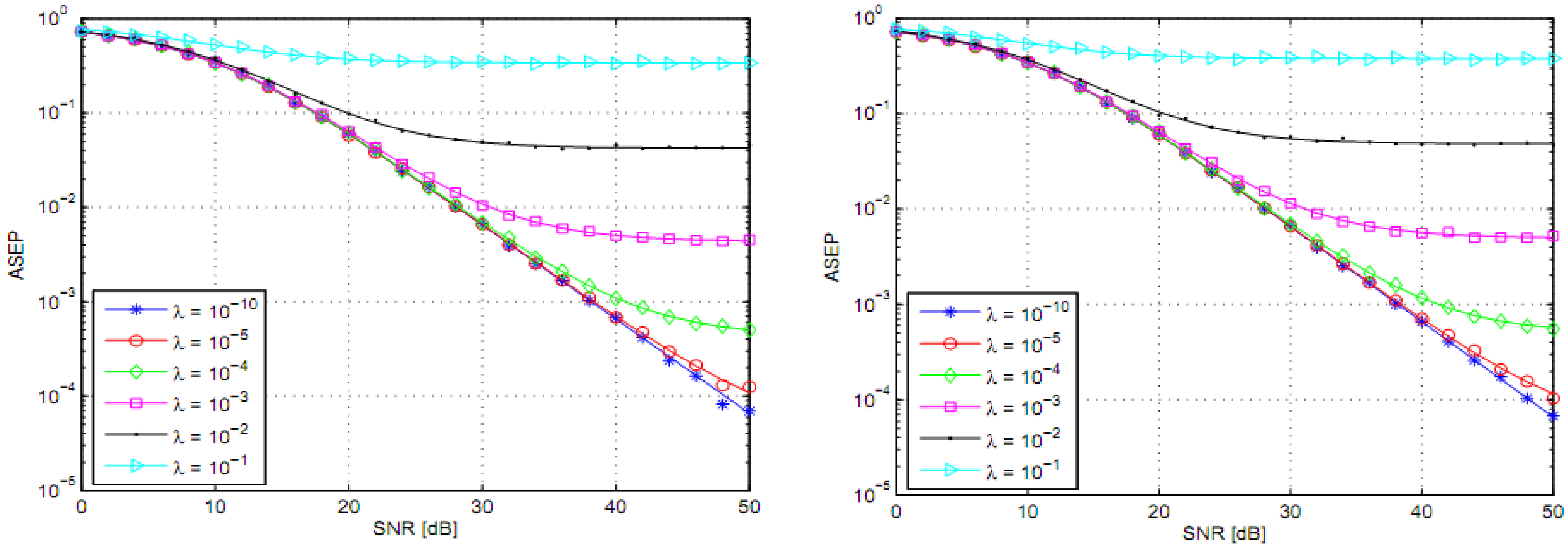
ASEP

- Pinto et al.: Semi-analytical framework
- Our result: single-integral expression not requiring Monte Carlo methods

$$\mathrm{ASEP} = \frac{M-1}{M}\pi - \frac{\sin\left(\frac{\pi}{M}\right)}{2\sqrt{\pi}} \int_0^\infty \left\{ \frac{1}{\sqrt{t}} \exp\left[-\left(\frac{\Psi_0}{\gamma_0} + \sin^2\left(\frac{\pi}{M}\right)\right) t \right] \right.$$
$$\left. \times \exp\left[-\left(\Psi_0 \sigma_I^2 \frac{\gamma_I}{\gamma_0} \right)^{\frac{1}{b_I}} t^{1/b_I} \right] \left[1 + \mathrm{erf}\left(\sqrt{t} \cos\left(\frac{\pi}{M}\right)\right)\right] \right\} \mathrm{d}t$$

- Simple expression
- Valid for synchronous and asynchronous networks
- All the main parameters are explicitly shown

ASEP



(a) Asynchronous (b) Synchronous

- Heavily interference-limited
- Synchronous results provide a tight upper bound for the asynchronous case

Conclusions and future works

- The proposed frameworks, substantiated by extensive Monte Carlo simulation, provide simple expressions of
  - Coverage Probability
  - Average Rate
  - Average Symbol Error Probability
- This permits to properly model network interference in next-generation networks (CRs, femto-cells, etc.)
- Future works include
  - Coverage over Nakagami-m fading channels
  - Rate over Lognormal channels
  - Extension to satellite-terrestrial systems

THANK YOU!